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Abstract

In this article we substantially extend the classification of combinatorial 3-manifolds with cyclic automorphism group up to 22 vertices. Moreover, several combinatorial criteria are given to decide, whether a cyclic combinatorial d-manifold can be generalized to an infinite family of such complexes together with a construction principle in the case that such a family exist. In addition, a new infinite series of cyclic neighborly combinatorial lens spaces of infinitely many distinct topological types is presented.

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Keywords: combinatorial 3-manifold, cyclic automorphism group, fundamental group, simplicial complexes, difference cycles, lens spaces

1 Introduction

An *abstract simplicial complex* C can be seen as a combinatorial structure consisting of tuples of elements of $\mathbb{Z}_n$ where the elements of $\mathbb{Z}_n$ are referred to as the vertices of the complex (cf. [13]). The *automorphism group* $\mathrm{Aut}(C)$ of C is the group of all permutations $\sigma \in S_n$ which do not change C as a whole. If $\mathrm{Aut}(C)$ acts transitively on the vertices, C is called a *transitive* simplicial complex. The most basic types of transitive simplicial complexes are the ones which are invariant under the cyclic $\mathbb{Z}_n$-action $v \mapsto v+1 \mod n$, i. e. all complexes C, such that $\mathbb{Z}_n$ is a subgroup of $\mathrm{Aut}(C)$. Such complexes are called *cyclic* simplicial complexes.

Many types of cyclic combinatorial structures have been investigated under several different aspects of combinatorics (see for example [14, Part V] for a work on cyclic Steiner systems in the field of design theory). This article is written in the context of combinatorial topology. Hence, we will concentrate on combinatorial manifolds, a special class of simplicial complexes, which are defined as follows: An abstract simplicial complex M is said to be *pure*, if all of its tuples are of length $d+1$, where d is referred to as the *dimension* of M. If, in addition, any vertex link of M, i. e. the boundary of a simplicial neighborhood of a vertex of M, is a triangulated $(d-1)$-sphere endowed with the standard piecewise linear structure, M is called a *combinatorial d-manifold*. There are several articles about cyclic combinatorial d-manifolds, see [12, 17] for many examples and further references.

One major advantage when dealing with simplicial complexes with large automorphism groups is that the complexes can be described efficiently just by the generators of its automorphism group and a system of orbit representatives of the complex under the group action. In the case of a cyclic automorphism group, the situation is particularly convenient. Since, possibly after a relabeling of the vertices, the whole complex does not change under a vertex-shift of type $v \mapsto v+1 \mod n$, two tuples are in one orbit if and only if the differences modulo n of its vertices are equal. Hence, we can compute a system of orbit representatives by just looking at the differences modulo n of the vertices of all tuples of the complex. This motivates the following definition.

Definition 1.1 (Difference cycle)**.** Let $a_i \in \mathbb{N}$, $0 \le i \le d$, $n := \sum_{i=0}^{d} a_i$ and $\mathbb{Z}_n = \langle (0,1,\dots,n-1) \rangle$. The simplicial complex

$$(a_0 : \dots : a_d) := \mathbb{Z}_n \langle 0, a_0, \dots, \Sigma_{i=0}^{d-1} a_i \rangle$$

is called *difference cycle of dimension d on n vertices* where $G\langle\cdot\rangle$ denotes the G-orbit of $\langle\cdot\rangle$. The number of elements of $(a_0 : \dots : a_d)$ is referred to as the *length* of the difference cycle. If a complex C is a union of difference cycles of dimension d on n vertices and λ is a unit of $\mathbb{Z}_n$ such that the complex λC (obtained by multiplying all vertex labels modulo n by λ) equals C, then λ is called a *multiplier* of C.

Note that for any unit $\lambda \in \mathbb{Z}_n^\times$, the complex λC is combinatorially isomorphic to C. In particular, all $\lambda \in \mathbb{Z}_n^\times$ are multipliers of the complex $\bigcup_{\lambda \in \mathbb{Z}_n^\times} \lambda C$ by construction. The definition of a difference cycle above is equivalent to the one given in [13].

In the following, we will describe cyclic simplicial complexes and cyclic combinatorial manifolds as a set of difference cycles. In this way, a lot of problems dealing with cyclic combinatorial manifolds can be solved in an elegant way. In particular, they play an important role in most of the proofs presented in this article.

Most calculations presented in this work were done with the help of a computer. In particular, the `GAP`-package `simpcomp` [6, 5, 7] as well as `GAP` [8] itself was used to handle difference cycles, permutation groups and quotients of free groups.

2 Classification of cyclic 3-manifolds

Neighborly combinatorial 3-manifolds with dihedral automorphism with up to 19 vertices as well as neighborly combinatorial 3-manifolds with cyclic automorphism group with up to 14 vertices have already been classified by Kühnel and Lassmann in 1985, see [12]. More recently, a more general classification of all transitive combinatorial manifolds with up to 13 vertices and all transitive combinatorial d-manifolds with $d \in \{2,3,9,10,11,12\}$ and up to 15 vertices was presented by Lutz in [17]. All classifications are based on an algorithm first described in [12]. As of Version 1.3, the classification algorithm is also available within `simpcomp`. This allows to extend any kind of classification of transitive simplicial complexes without the need for any further programming.

In a series of computer calculations, we computed all cyclic combinatorial 3-manifolds with up to 22 vertices. This led to the following result.

Theorem 2.1 (Classification of cyclic combinatorial 3-manifolds)**.** *There are exactly* 59519 *(connected) combinatorial* 3*-manifolds with cyclic automorphism group with up to* 22 *vertices. These complexes split up into* 6070 *combinatorial types and at least* 54 *topological types.*

In particular, we have triangulations of the following topological 3-manifolds:

1. *The* 3*-sphere* S^3. *The smallest cyclic triangulation is the boundary of the* 4*-simplex*

$$\partial\Delta^4 = \{(1:1:1:2)\}.$$

2. *The* 3*-dimensional Klein bottle* $S^2 \mathbin{\underline{\times}} S^1$. *The smallest cyclic triangulation is the minimal and tight* 9*-vertex triangulation first described by Altshuler and Steinberg in [2, Complex* N^9_{51}*], given by the difference cycles*

$$\{(1:1:2:5),(1:1:5:2),(1:2:1:5)\}.$$

3. *The orientable* 3*-dimensional sphere bundle* $S^2 \times S^1$. *The smallest cyclic triangulation is the minimal* 10*-vertex triangulation first described by Kühnel and Lassmann [13, Complex* $M^3_2(10)$*] as a generalization of Altshuler and Steinberg's* 9*-vertex* 3*-dimensional Klein bottle, given by the difference cycles*

$$\{(1:1:2:6),(1:1:6:2),(1:2:1:6)\}.$$

4. *The twofold connected sum of the orientable* 3*-dimensional sphere bundle* $(S^2\times S^1)^{\#2}$. *The smallest cyclic triangulation is the minimal* 12*-vertex triangulation first described by Kühnel and Lassmann in [12, Complex* 5_{12}*].*

$$\{(1:2:3:6),(1:2:4:5),(1:5:1:5),(2:2:2:6),(2:3:3:4)\}$$

5. *A lens space of type* $L(3,1)$. *The smallest cyclic triangulation is the minimal* 14*-vertex triangulation first described by Kühnel and Lassmann in [12, Complex* 3_{14}*]. For an alternative proof of its topological type see Theorem 4.1.*

$$\{(1:1:1:11),(1:2:4:7),(1:4:2:7),(1:4:7:2),(2:4:4:4),(2:5:2:5)\}$$

6. *The real projective* 3*-space* $\mathbb{R}P^3$. *The smallest cyclic complex has* 15 *vertices and was first described by Kühnel and Lassmann in [12, Complex* 2_{15}*].*

$$\{(1:1:1:12),(1:2:3:9),(1:5:7:2),(2:3:3:7),(3:4:3:5),(3:4:4:4)\}$$

7. *The prism manifold or cube space** $P_2 = S^3/Q_8$, *where the fundamental group* Q_8 *denotes the quaternion group of order* 8. *The smallest cyclic complex has* 15 *vertices and was first described by Kühnel and Lassmann in [12, Complex* 8_{15}*].*

$$\{(1:1:1:12),(1:2:4:8),(1:6:6:2),(2:4:3:6),(3:4:4:4)\}$$

8. *The* 3*-torus* $\mathbb{T}^{3***}$. *The smallest cyclic complex has* 15 *vertices and is* locally minimal, *i. e. it cannot be reduced by bistellar moves without inserting additional vertices first. The complex was first described by Kühnel and Lassmann in [12, Complex* III_{15}*].*

$$\{(1:2:4:8),(1:2:8:4),(1:4:2:8),(1:4:8:2),(1:8:2:4),(1:8:4:2)\}$$

9. *The flat manifold* $\mathfrak{B}_2{}^{***}$ *with fundamental group*

$$\langle a, b \mid ab^2 = b^2a, a^2b = ba^2 \rangle.$$

The smallest cyclic complex is centrally symmetric, has 16 *vertices and is due to Lutz in [16, Complex* ${}^3 16^{55}_{10}$*]. The complex was first described in [17, p. 89] where no proof of its topological type was given.*

$$\{(1:1:3:11),(1:1:4:10),(1:3:2:10),(2:3:4:7),(2:4:7:3),(2:7:3:4)\}$$

10. *The spherical manifold* $S^3/\mathrm{SL}(2,3)$ *of tetrahedral type* with fundamental group* $\mathrm{SL}(2,3)$ *(the binary tetrahedral group) of order* 24*. The smallest cyclic complex has* 16 *vertices and was first described by Lutz in [16, Complex* ${}^3 16^{1}_{31}$*].*

$$\{(1:1:3:11),(1:1:4:10),(1:3:2:10),(2:3:8:3),(2:4:6:4),(3:5:3:5)\}$$

11. *The connected sum* $(S^2 \times S^1)^{\#5}$*. The smallest cyclic complex has* 16 *vertices and can be found in [16, Complex* ${}^3 16^{1}_{41}$*].*

$$\{(1:2:5:8),(1:2:6:7),(1:3:4:8),(1:3:5:7),(2:5:3:6),(2:6:2:6),(3:4:4:5)\}$$

12. *The Poincaré homology sphere** Σ^3 *with fundamental group* $\mathrm{SL}(2,5)$*. The smallest cyclic complex has* 17 *vertices and can be found in [16, Complex* ${}^3 17^{1}_{21}$*].*

$$\{(1:1:1:14),(1:2:4:10),(1:6:8:2),(2:3:4:8),(2:3:6:6),(2:4:5:6),(4:4:4:5)\}$$

13. *The* S^1*-bundle over the real projective plane* $S^1 \times \mathbb{R}P^2$ *of Heegaard genus* 2*. The smallest cyclic complex has* 17 *vertices, is locally minimal and was first described by Kühnel and Lassmann (see [12, Complex* IV_{17}*]) and identified by Lutz (see [16, Complex* ${}^3 17^{2}_{13}$*]).*

$$\{(1:1:1:14),(1:2:12:2),(2:3:9:3),(3:4:4:6),(3:4:6:4),(3:5:3:6),(3:6:4:4)\}$$

14. *A lens space of type* $L(8,3)$*. The smallest cyclic complex has* 18 *vertices. For a proof of its topological type see Theorem 4.1.*

$$\{(1:1:1:15),(1:2:4:11),(1:4:2:11),(1:4:11:2),(2:4:8:4),(2:5:2:9),(2:7:2:7),(4:4:4:6)\}$$

15. *A non-orientable manifold* M_{15} *with homology groups* $(\mathbb{Z}, \mathbb{Z}_2^2 \oplus \mathbb{Z}, \mathbb{Z}_2, 0)$ *and Heegaard genus* 3*. The smallest cyclic complex has* 18 *vertices and is locally minimal.*

$$\{(1:1:1:15),(1:2:5:10),(1:4:3:10),(1:4:11:2),(3:4:5:6),(3:5:6:4),(3:6:3:6),(3:6:4:5)\}$$

16. *A lens space of type* $L(5,1)$*. The smallest cyclic complex has* 18 *vertices.*

$$\{(1:1:1:15),(1:2:5:10),(1:5:2:10),(1:5:10:2),(2:5:2:9),(2:6:4:6),(2:7:2:7),(4:4:4:6)\}$$

17. *A triangulation of* $\mathbb{K}^2 \times S^{1***}$ *with fundamental group*

$$\langle a, b, c \mid ab = ba, ac = ca, bcb = c \rangle.$$

The smallest cyclic complex has 18 *vertices.*

$$\{(1:1:3:13),(1:1:6:10),(1:3:1:13),(1:6:8:3),(1:7:6:4),(2:3:7:6),(2:6:4:6)\}$$

18. *The flat manifold* $\mathfrak{B}_4{}^{***}$ *with fundamental group*

$$\langle a, b \mid ab^2 = b^2a, b = a^2ba^2 \rangle.$$

The smallest cyclic complex has 18 *vertices and is locally minimal.*

$\{(1:1:3:13), (1:1:13:3), (1:3:1:13), (2:3:6:7), (2:6:2:8), (2:6:7:3), (2:7:2:7), (2:7:3:6)\}$

19. *The connected sum* $(S^2 \times S^1)^{\#7}$*. The smallest cyclic complex has* 18 *vertices.*

$\{(1:1:7:9), (1:1:8:8), (1:7:2:8), (2:3:4:9), (2:3:6:7), (3:3:3:9), (3:4:5:6), (4:5:4:5)\}$

20. *The connected sum* $(S^2 \underline{\times} S^1)^{\#7}$*. The smallest cyclic complex has* 18 *vertices.*

$\{(1:1:7:9), (1:1:9:7), (1:7:1:9), (2:3:4:9), (2:3:6:7), (3:3:3:9), (3:4:5:6), (4:5:4:5)\}$

21. *A manifold* M_{21} *with homology group* $(\mathbb{Z}, \mathbb{Z}_4 \oplus \mathbb{Z}, \mathbb{Z}, \mathbb{Z})$ *of Heegaard genus at least* 2 *and at most* 3*. The smallest cyclic complex has* 20 *vertices.*

$\{(1:1:1:17), (1:2:4:13), (1:6:8:5), (1:8:6:5), (1:8:9:2), (2:4:5:9), (3:4:4:9), (4:4:5:7)\}$

22. *A non-orientable manifold* M_{22} *with homology groups* $(\mathbb{Z}, \mathbb{Z}_3 \oplus \mathbb{Z}, \mathbb{Z}_2, 0)$ *of Heegaard genus* 2*. The smallest cyclic complex has* 20 *vertices.*

$\{(1:1:1:17), (1:2:5:12), (1:5:2:12), (1:5:12:2), (2:5:4:9), (2:6:6:6), (2:9:4:5), (4:5:4:7)\}$

23. *A prism manifold** P_7*, determined by its fundamental group* $D_7 \times \mathbb{Z}_2$ *of order* 28*. The smallest cyclic complex has* 20 *vertices and is locally minimal.*

$\{(1:1:1:17), (1:2:15:2), (2:3:12:3), (3:4:6:7), (3:4:7:6), (3:5:3:9), (3:6:3:8), (3:6:4:7), (4:6:4:6)\}$

24. *The connected sum* $(S^2 \times S^1)^{\#6}$*. The smallest cyclic complex has* 20 *vertices.*

$\{(1:1:3:15), (1:1:4:14), (1:3:5:11), (1:5:5:9), (1:8:2:9), (2:3:7:8), (2:4:5:9), (3:5:5:7), (3:7:3:7)\}$

25. *The flat manifold* $\mathfrak{G}_2{}^{***}$ *with fundamental group*

$$\langle a, b, c \mid aba = b, cbc = b, ac = ca \rangle.$$

The smallest cyclic complex has 20 *vertices.*

$\{(1:1:3:15), (1:1:6:12), (1:3:1:15), (1:6:10:3), (1:7:2:10), (1:9:6:4), (2:3:7:8), (2:6:5:7), (4:6:4:6)\}$

26. *The connected sum* $(S^2 \underline{\times} S^1)^{\#6}$*. The smallest cyclic complex has* 20 *vertices.*

$\{(1:1:3:15), (1:1:8:10), (1:3:7:9), (1:4:6:9), (1:8:5:6), (1:9:4:6), (2:3:5:10), (3:5:5:7), (3:7:3:7)\}$

27. *The connected sum* $(S^2 \times S^1)^{\#4}$*. The smallest cyclic complex has* 20 *vertices.*

$\{(1:2:2:15), (1:2:4:13), (1:4:5:10), (1:6:4:9), (1:9:1:9), (2:2:4:12), (2:6:9:3), (3:4:4:9), (4:5:5:6)\}$

28. *The connected sum* $(S^2 \underline{\times} S^1)^{\#9}$. *The smallest cyclic complex has* 20 *vertices.*

$\{(1:2:7:10),(1:2:8:9),(1:4:5:10),(1:4:11:4),(1:10:5:4),(2:6:2:10),(2:6:6:6),(2:7:3:8),(3:7:3:7)\}$

29. *The flat manifold* $\mathfrak{G}_3{}^{***}$ *with fundamental group*

$$\langle a, b \mid (ba)^2 = a^{-1}b^2, (ba^{-1})^2 = ab^2 \rangle.$$

The smallest cyclic complex has 21 *vertices.*

$\{(1:1:1:18),(1:2:1:17),(1:3:5:12),(1:5:3:12),(1:5:6:9),(1:11:6:3),(3:5:8:5),(4:5:6:6)\}$

30. *A manifold* M_{30} *with homology groups* $(\mathbb{Z}, \mathbb{Z}_2 \oplus \mathbb{Z}^3, \mathbb{Z}_2 \oplus \mathbb{Z}^2, 0)$ *of Heegaard genus* 4. *The smallest cyclic complex has* 21 *vertices.*

$\{(1:1:1:18),(1:2:1:17),(1:3:6:11),(1:6:3:11),(1:6:11:3),(3:5:6:7),(3:5:8:5),(3:6:7:5),(3:7:5:6)\}$

31. *A manifold* M_{31} *with homology groups* $(\mathbb{Z}, \mathbb{Z}_3 \oplus \mathbb{Z}_6 \oplus \mathbb{Z}, \mathbb{Z}_2, 0)$ *of Heegaard genus* 3. *The smallest cyclic complex has* 21 *vertices.*

$\{(1:1:1:18),(1:2:5:13),(1:4:3:13),(1:4:14:2),(3:4:3:11),(3:5:6:7),(3:6:6:6),(3:6:7:5),(3:7:5:6)\}$

32. *A manifold* M_{32} *with homology groups* $(\mathbb{Z}, \mathbb{Z}_3^2 \oplus \mathbb{Z}, \mathbb{Z}_2, 0)$ *of Heegaard genus* 3. *The smallest cyclic complex has* 21 *vertices.*

$\{(1:1:1:18),(1:2:6:12),(1:4:4:12),(1:4:14:2),(2:5:4:10),(2:6:3:10),(3:4:10:4),(3:6:6:6),(3:10:4:4)\}$

33. *A manifold* M_{33} *with homology groups* $(\mathbb{Z}, \mathbb{Z}_2^2 \oplus \mathbb{Z}^2, \mathbb{Z} \oplus \mathbb{Z}_2, 0)$ *of Heegaard genus* 4. *The smallest cyclic complex has* 21 *vertices and is locally minimal.*

$\{(1:2:3:15),(1:2:13:5),(1:5:2:13),(1:7:4:9),(1:11:4:5),(2:3:3:13),(2:6:4:9),(2:10:4:5),(4:6:4:7)\}$

34. *A manifold* M_{34} *with homology groups* $(\mathbb{Z}, \mathbb{Z}_7 \oplus \mathbb{Z}^2, \mathbb{Z}^2, \mathbb{Z})$ *of Heegaard genus* 3. *The smallest cyclic complex has* 21 *vertices.*

$\{(1:2:4:14),(1:2:5:13),(1:6:5:9),(1:7:9:4),(1:11:3:6),(1:14:2:4),(3:4:3:11),(3:5:9:4),(3:6:5:7)\}$

35. *The connected sum* $(S^2 \times S^1)^{\#12}$. *The smallest cyclic complex has* 21 *vertices.*

$\{(1:2:4:14),(1:2:11:7),(1:6:3:11),(1:9:4:7),(2:4:7:8),(3:3:3:12),(3:4:5:9),(3:6:7:5),(3:9:4:5)\}$

36. *The prism manifold** $P_8 = S^3/Q_{32}$, *determined by its fundamental group* Q_{32} *which denotes the generalized quaternion group of order* 32. *The smallest cyclic complex has* 22 *vertices.*

$\{(1:1:1:19),(1:2:1:18),(1:3:15:3),(3:4:3:12),(3:5:6:8),(3:5:8:6),(3:6:3:10),(3:6:5:8),(3:7:3:9),(5:6:5:6)\}$

37. *The prism manifold** $P_4 = S^3/Q_{16}$, *determined by its fundamental group* Q_{16} *which denotes the generalized quaternion group of order* 16. *The smallest cyclic complex has* 22 *vertices.*

$\{(1:1:1:19),(1:2:3:16),(1:5:7:9),(1:12:7:2),(2:3:3:14),(2:6:7:7),(3:4:8:7),(3:4:10:5),(4:4:4:10)\}$

38. *A lens space of type* $L(15,4)$*. The smallest cyclic complex has* 22 *vertices. For a proof of its topological type see Theorem 4.1.*

$\{(1:1:1:19),(1:2:4:15),(1:4:2:15),(1:4:15:2),(2:4:12:4),(2:5:2:13),(2:7:2:11),(2:9:2:9),(4:4:4:10),(4:6:4:8)\}$

39. *A manifold* M_{39} *with homology groups* $(\mathbb{Z},\mathbb{Z},\mathbb{Z},\mathbb{Z})$ *and a fundamental group different to* $\mathbb{Z}$*. The smallest cyclic complex has* 22 *vertices.*

$\{(1:1:1:19),(1:2:5:14),(1:7:12:2),(2:4:5:11),(2:4:11:5),(2:5:4:11),(2:8:2:10),(2:12:3:5),(4:5:4:9),(5:6:5:6)\}$

40. *A lens space of type* $L(7,1)$*. The smallest cyclic complex has* 22 *vertices. For a proof of its topological type see below.*

$\{(1:1:1:19),(1:2:5:14),(1:7:12:2),(2:5:2:13),(2:7:2:11),(2:8:4:8),(2:9:2:9),(2:12:3:5),(4:6:4:8),(4:6:6:6)\}$

41. *A manifold* M_{41} *with homology groups* $(\mathbb{Z},\mathbb{Z}_5\oplus\mathbb{Z},\mathbb{Z}_2,0)$ *of Heegaard genus* 2*. The smallest cyclic complex has* 22 *vertices and is locally minimal.*

$\{(1:1:5:15),(1:1:15:5),(1:5:1:15),(2:3:2:15),(2:3:8:9),(2:8:4:8),(2:8:9:3),(2:9:2:9),(2:9:3:8),(4:4:4:10)\}$

42. *A manifold* M_{42} *with homology groups* $(\mathbb{Z},\mathbb{Z}^3,\mathbb{Z}_2\oplus\mathbb{Z}^2,0)$ *of Heegaard genus* 3*, different to* $(S^2 \underline{\times} S^1)^{\#3}$*. The smallest cyclic complex has* 22 *vertices and is locally minimal.*

$\{(1:1:5:15),(1:1:15:5),(1:5:1:15),(2:5:3:12),(2:8:4:8),(2:9:2:9),(2:9:3:8),(2:11:4:5),(3:8:4:7),(3:10:4:5)\}$

43. *The connected sum* $(S^2 \underline{\times} S^1)^{\#12}$*. The smallest cyclic complex has* 22 *vertices.*

$\{(1:1:9:11),(1:1:10:10),(1:9:2:10),(2:3:6:11),(2:3:8:9),(3:4:4:11),(3:4:11:4),(3:6:5:8),(3:11:4:4),(5:6:5:6)\}$

44. *A manifold* M_{44} *with homology groups* $(\mathbb{Z},\mathbb{Z}_4\oplus\mathbb{Z}^2,\mathbb{Z}_2\oplus\mathbb{Z},0)$ *of Heegaard genus* 3*. The smallest cyclic complex has* 22 *vertices.*

$\{(1:2:4:15),(1:2:13:6),(1:4:2:15),(1:4:8:9),(1:12:3:6),(2:4:8:8),(2:12:3:5),(2:13:3:4),(3:5:8:6)\}$

45. *A homology sphere* M_{45} *with unknown but very large*** *fundamental group of Heegaard genus* 2*. The smallest cyclic complex has* 19 *vertices.*

$\{(1:1:1:16),(1:2:6:10),(1:8:8:2),(2:6:3:8),(3:6:4:6),(4:5:4:6),(4:5:5:5)\}$

46. *A manifold* M_{46} *with homology groups* $(\mathbb{Z},\mathbb{Z}_3,0,\mathbb{Z})$ *and unknown but very large*** *fundamental group of Heegaard genus* 2*. The smallest cyclic complex has* 20 *vertices.*

$\{(1:1:5:13),(1:1:6:12),(1:5:2:12),(2:5:2:11),(2:6:6:6),(2:7:4:7),(3:4:3:10),(3:4:9:4),(3:7:3:7)\}$

47. *A manifold* M_{47} *with homology groups* $(\mathbb{Z},\mathbb{Z}_3^2,0,\mathbb{Z})$ *and unknown but very large*** *fundamental group of Heegaard genus at least* 2 *and at most* 3*. The smallest cyclic complex has* 20 *vertices.*

$\{(1:1:3:15),(1:1:4:14),(1:3:4:12),(1:5:2:12),(2:3:6:9),(2:4:9:5),(2:9:3:6),(3:4:4:9)\}$

48. *A manifold* M_{48} *with homology groups* $(\mathbb{Z},\mathbb{Z}_2^2,0,\mathbb{Z})$ *and unknown but very large*** *fundamental group of Heegaard genus* 2*. The smallest cyclic complex has* 21 *vertices.*

$\{(1:1:1:18),(1:2:7:11),(1:9:9:2),(2:7:3:9),(3:7:4:7),(4:6:4:7),(4:6:5:6),(5:5:5:6)\}$

49. *A manifold M_{49} with homology groups $(\mathbb{Z}, \mathbb{Z}_3 \oplus \mathbb{Z}_9, 0, \mathbb{Z})$ and unknown but very large** fundamental group of Heegaard genus 2. The smallest cyclic complex has 21 vertices.*

$\{(1:1:1:18),(1:2:1:17),(1:3:6:11),(1:6:3:11),(1:6:11:3),(3:5:3:10),(3:5:8:5),(3:6:6:6),(3:7:3:8)\}$

50. *A manifold M_{50} with homology groups $(\mathbb{Z}, \mathbb{Z}_5 \oplus \mathbb{Z}_{10}, 0, \mathbb{Z})$ and unknown but very large** fundamental group of Heegaard genus 2. The smallest cyclic complex has 21 vertices and is locally minimal.*

$\{(1:1:3:16),(1:1:4:15),(1:3:10:7),(1:5:8:7),(2:3:6:10),(2:4:10:5),(2:6:3:10),(2:6:8:5),(3:6:3:9)\}$

51. *A manifold M_{51} with homology groups $(\mathbb{Z}, \mathbb{Z}_4^2, 0, \mathbb{Z})$ and unknown but very large** fundamental group of Heegaard genus 2. The smallest cyclic complex has 21 vertices and is locally minimal.*

$\{(1:1:3:16),(1:1:4:15),(1:3:10:7),(1:5:8:7),(2:3:6:10),(2:4:10:5),(2:6:3:10),(2:6:8:5),(3:6:6:6)\}$

52. *A manifold M_{52} with homology groups $(\mathbb{Z}, \mathbb{Z}_5, 0, \mathbb{Z})$ and unknown but very large** fundamental group of Heegaard genus 2. The smallest cyclic complex has 21 vertices.*

$\{(1:1:1:19),(1:2:17:2),(2:3:4:13),(2:7:10:3),(3:4:8:7),(3:5:4:10),(3:5:7:7),(4:6:4:8),(4:6:6:6)\}$

53. *A manifold M_{53} with homology groups $(\mathbb{Z}, \mathbb{Z}_8, 0, \mathbb{Z})$ and unknown but very large** fundamental group of Heegaard genus 2. The smallest cyclic complex has 21 vertices.*

$\{(1:1:3:17),(1:1:4:16),(1:3:2:16),(2:3:14:3),(2:4:12:4),(3:5:3:11),(3:8:3:8),(4:6:6:6)\}$

54. *A manifold M_{54} with homology groups $(\mathbb{Z}, \mathbb{Z}_{24}, 0, \mathbb{Z})$ and unknown but very large** fundamental group of Heegaard genus at least 2 and at most 3. The smallest cyclic complex has 22 vertices and is locally minimal.*

$\{(1:1:3:17),(1:1:4:16),(1:3:2:16),(2:3:7:10),(2:4:2:14),(2:6:8:6),(2:7:2:11),(2:7:10:3),(2:9:2:9),(2:10:3:7)\}$

** *"very large" in this context means that* ***GAP*** *wasn't able to calculate the size of the fundamental group due to extremely large orders of its generators. In particular, the fundamental group might be of infinite order.*

Proof. The complexes were found using the classification algorithm for transitive combinatorial manifolds integrated to the software package simpcomp.

Most of the topological distinctions were done via comparison of the simplicial homology groups and the fundamental group of the complexes:

- The manifolds of type $(S^2 \times S^1)^{\#k}$ and $(S^2 \underline{\times} S^1)^{\#k}$ were identified by calculating the fundamental group and applying Kneser's conjecture, proved by Stallings in 1959 (see [25]) together with [9, Theorem 5.2].
- By the elliptization conjecture (stated by Thurston in [27, Chapter 3], recently proved by Perelman, see [20, 22, 21]), the topological type of a spherical 3-manifold distinct from a lens space is already determined by the isomorphism type of its (finite) fundamental group. These cases are marked by * in the list above.

- The fundamental group distinguishes all flat 3-manifolds by a theorem of Bieberbach (see [3] and [18, page 4]). On the other hand, all other 3-manifolds with a fundamental group containing $\mathbb{Z}^3$ are known to be the connected sum of a flat 3-manifold with some other 3-manifold (cf. [15]). Hence, all 3-manifolds with the fundamental group of a flat manifold have to be prime (as all flat manifolds are prime and the fundamental group of some 3-manifold M determines the length of a prime decomposition of M, cf. [25] and [9, Theorem 5.2]) and thus are flat. Altogether, a 3-manifold with the fundamental group of a flat manifold is determined by its fundamental group. These cases are marked by *** in the list above.

For more information about the spherical case in the classification of 3-manifolds see [26, 19], for more about flat 3-manifolds see [3, 18, 11].

Now let us prove that complex *16* - which will be denoted by C in the following - is homeomorphic to the lens space $L(5,1)$:

Figure 2.1 shows the *slicing*, i. e. the pre-image of a polyhedral Morse function or *regular simplexwise linear function* (see [10]) as described in [24], of C between the odd labeled vertices and the even labeled vertices. Here, the slicing is a torus. Also, both the span of the odd and the span of the even labeled vertices is a solid torus and hence C is a manifold of Heegaard genus 1. For the 1-homology of the two tori $T_- := \partial(\text{span}(0,2,\ldots,16))$ and $T_+ := \partial(\text{span}(1,3,\ldots,17))$ we choose a basis as follows:

$$\begin{aligned} \alpha_- &:= \langle 0,10,4,14,8,0 \rangle \\ \beta_- &:= \langle 0,12,6,0 \rangle \end{aligned}$$

and

$$\begin{aligned} \alpha_+ &:= \langle 1,11,5,15,9,1 \rangle \\ \beta_+ &:= \langle 1,13,7,1 \rangle \end{aligned}$$

such that $H_1(T_\pm) = \langle \alpha_\pm, \beta_\pm \rangle$, $H_1(\text{span}(0,2,\ldots,16)) = \langle \beta_- \rangle$ and $H_1(\text{span}(1,3,\ldots,17)) = \langle \beta_+ \rangle$.

Now, we want to express α_- in terms of α_+ and β_+. With the help of the slicing (the thick line in Figure 2.1 denotes a path homologous to α_- in the slicing) we see that α_- can be transported to the path

$$\langle 17,15,7,5,3,13,11,3,1,17,9,7,17 \rangle$$

which entirely lies in T_+. This path is homologous to -5 times β_+ and 4 times α_+ and hence the topological type of C must be $L(-5,4) \cong L(5,1)$.

In the following, we will prove that complex *40* - which will be denoted by D - is homeomorphic to the lens space $L(7,1)$:

Figure 2.2 shows the slicing of D between the odd labeled vertices and the even labeled vertices which is a torus. Also, both the span of the odd and the span of the even labeled

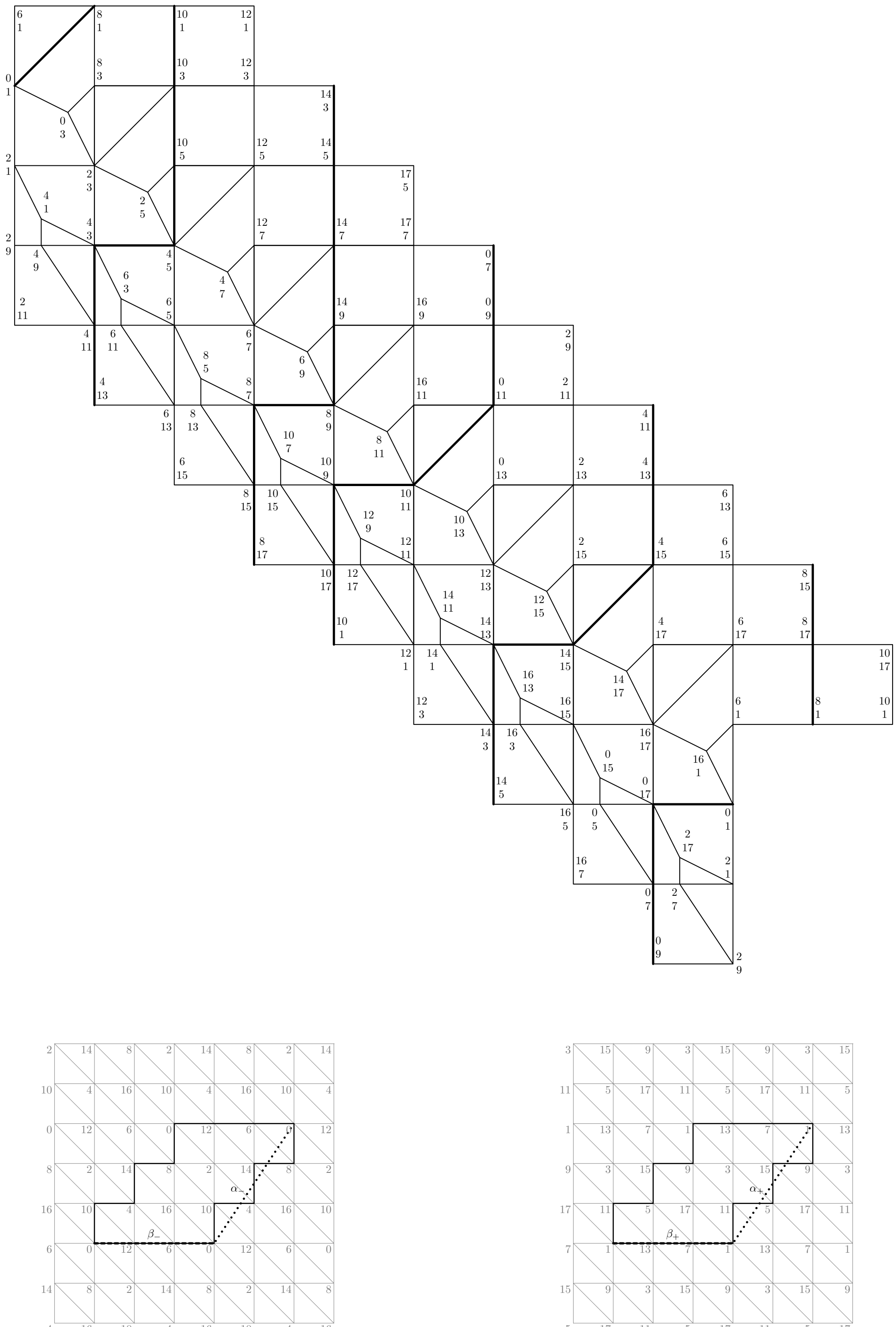


Figure 2.1: Slicing of C between the odd labeled and the even labeled vertices together with the boundary of the two solid tori spanned by the even and by the odd vertices.

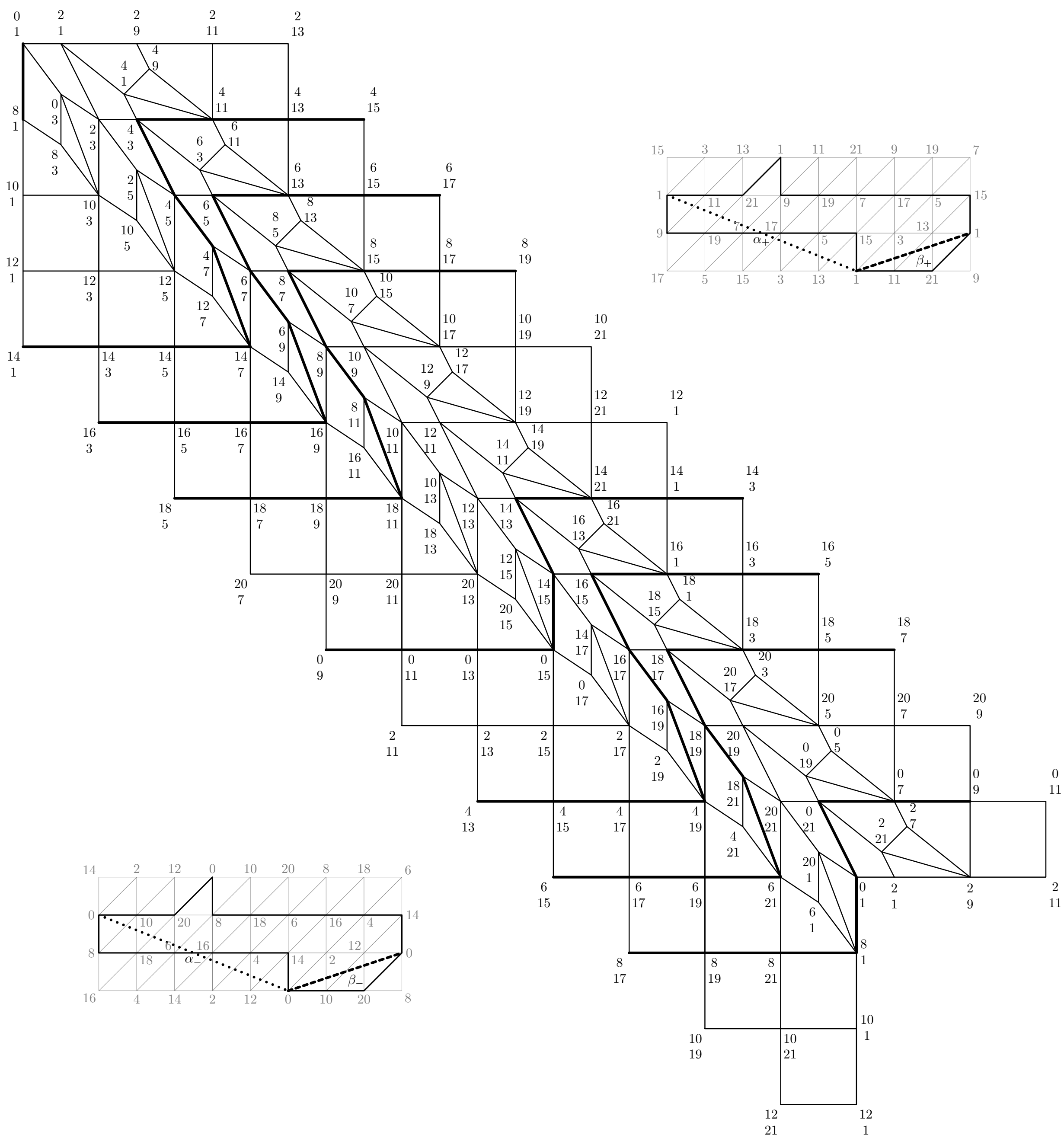


Figure 2.2: Slicing of D between the odd labeled and the even labeled vertices together with the boundary of the two solid tori spanned by the even and by the odd vertices.

vertices is a solid torus and hence D is a manifold of Heegaard genus 1. For the 1-homology of the two tori $T_- := \partial(\mathrm{span}(0,2,\ldots,16))$ and $T_+ := \partial(\mathrm{span}(1,3,\ldots,17))$ we choose a basis as follows:

$$\begin{aligned} \alpha_- &:= \langle 0,8,18,6,16,4,14,0\rangle \\ \beta_- &:= \langle 0,2,4,0\rangle \end{aligned}$$

and

$$\begin{aligned} \alpha_+ &:= \langle 1,9,19,7,17,5,15,1\rangle \\ \beta_+ &:= \langle 1,3,5,1\rangle \end{aligned}$$

such that $H_1(T_\pm) = \langle \alpha_\pm, \beta_\pm\rangle$, $H_1(\mathrm{span}(0,2,\ldots,16)) = \langle \beta_-\rangle$ and $H_1(\mathrm{span}(1,3,\ldots,17)) = \langle \beta_+\rangle$.

Once again, we want to express α_- in terms of α_+ and β_+. With the help of the slicing (the thick line in Figure 2.2 denotes a path homologous to α_- in the slicing) we see that α_- can be transported to the path

$$\langle 21,19,17,15,7,5,3,17,15,13,5,3,1,15,13,11,3,1,21,13,11,9,7,21\rangle$$

which entirely lies in T_+. This path is homologous to -7 times β_+ and -1 times α_+ and hence the topological type of D must be $L(-7,-1) \cong L(7,1)$.

For the identification of the exact topological type of the complexes number *5*, *14* and *38* see Theorem 4.1. For the complexes *1* – *13* see the indicated sources. The topological type of the other complexes has to be left open at this point.

The exact number of complexes, combinatorial types, homological types and locally minimal complexes, sorted by vertex numbers, can be found in Table 1.

A list of all occurring finite fundamental groups of cyclic 3-manifolds with 22 or less vertices which `GAP` was able to compute, is shown in Table 2. A list of all homological types of cyclic combinatorial 3-manifolds with 22 or less vertices is listed in Table 3. □

Remark 2.2. Note that there exist a lot of examples of topologically distinct 3-manifolds which cannot be distinguished by comparison of the homology groups or the Heegaard genus. In addition, as the fundamental group is given by the edge group which is a quotient of a free group, recognizing its isomorphism type is not always possible. Hence, we can expect the number of topologically distinct cyclic combinatorial 3-manifolds with 22 or less vertices to be larger than indicated in Theorem 2.1.

Table 1: The classification of cyclic combinatorial 3-manifolds with up to 22 vertices

n	# compl.	# dist. c.	# loc. min. c.	# dist. lmc.	# hom. types
5	1	1	1	1	1
6	1	1	0	0	1
7	3	1	0	0	1
8	3	2	0	0	1
9	6	2	3	1	2
10	19	8	0	0	3
11	40	6	0	0	2
12	56	20	0	0	4
13	135	15	0	0	2
14	258	50	0	0	4
15	217	34	1	1	5
16	742	107	12	2	7
17	1272	89	24	2	6
18	1818	319	24	4	15
19	4797	279	63	4	5
20	7670	1008	66	9	19
21	11931	1038	198	18	20
22	30550	3090	230	23	23

1st column: number of vertices n,
2nd column: number of cyclic combinatorial 3-manifolds,
3rd column: number of combinatorially distinct cyclic combinatorial 3-manifolds
4th column: number of locally minimal cyclic combinatorial 3-manifolds
5th column: number of combinatorially distinct locally minimal cyclic combinatorial 3-manifolds,
6th column: number of homological types of cyclic combinatorial 3-manifolds.

Table 2: Finite fundamental groups of cyclic combinatorial 3-manifolds.

fundamental group	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
C_{15}																		×
C_2											×		×	×			×	×
C_3										×		×				×	×	×
C_5														×				×
C_7																		×
$C_7 \rtimes C_4$																×		
C_8														×		×		×
Q_{16}																		×
Q_{32}																		×
Q_8											×			×			×	
SL(2, 3)												×						×
SL(2, 5)													×					×

Table 3: Homological types of cyclic combinatorial 3-manifolds.

homology groups	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
$(\mathbb{Z},0,0,\mathbb{Z})$	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
$(\mathbb{Z},\mathbb{Z}_2,0,\mathbb{Z})$											×		×	×			×	×
$(\mathbb{Z},\mathbb{Z}_2\oplus\mathbb{Z}_2,0,\mathbb{Z})$											×			×			×	×
$(\mathbb{Z},\mathbb{Z}_3,0,\mathbb{Z})$										×		×				×	×	×
$(\mathbb{Z},\mathbb{Z}_3\oplus\mathbb{Z}_3,0,\mathbb{Z})$																×		×
$(\mathbb{Z},\mathbb{Z}_3\oplus\mathbb{Z}_9,0,\mathbb{Z})$																	×	
$(\mathbb{Z},\mathbb{Z}_4,0,\mathbb{Z})$																×		
$(\mathbb{Z},\mathbb{Z}_4\oplus\mathbb{Z}_4,0,\mathbb{Z})$																	×	
$(\mathbb{Z},\mathbb{Z}_5,0,\mathbb{Z})$														×				×
$(\mathbb{Z},\mathbb{Z}_5\oplus\mathbb{Z}_{10},0,\mathbb{Z})$																	×	
$(\mathbb{Z},\mathbb{Z}_7,0,\mathbb{Z})$																		×
$(\mathbb{Z},\mathbb{Z}_8,0,\mathbb{Z})$														×		×		×
$(\mathbb{Z},\mathbb{Z}_{15},0,\mathbb{Z})$																		×
$(\mathbb{Z},\mathbb{Z}_{24},0,\mathbb{Z})$																		×
$(\mathbb{Z},\mathbb{Z},\mathbb{Z}_2,0)$					×	×	×	×	×	×	×	×	×	×	×	×	×	×
$(\mathbb{Z},\mathbb{Z},\mathbb{Z},\mathbb{Z})$						×		×		×		×		×		×		×
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_2,\mathbb{Z}_2,0)$													×		×	×	×	×
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2,\mathbb{Z}_2,0)$														×			×	
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2,\mathbb{Z},\mathbb{Z})$																×		
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_3,\mathbb{Z}_2,0)$																×		×
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_3,\mathbb{Z},\mathbb{Z})$																	×	
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_3\oplus\mathbb{Z}_3,\mathbb{Z}_2,0)$																	×	
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_3\oplus\mathbb{Z}_6,\mathbb{Z}_2,0)$																	×	
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_4,\mathbb{Z}_2,0)$														×		×		×
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_4,\mathbb{Z},\mathbb{Z})$																×		
$(\mathbb{Z},\mathbb{Z}\oplus\mathbb{Z}_5,\mathbb{Z}_2,0)$																		×
$(\mathbb{Z},\mathbb{Z}^2,\mathbb{Z}\oplus\mathbb{Z}_2,0)$												×	×	×	×	×	×	×
$(\mathbb{Z},\mathbb{Z}^2,\mathbb{Z}^2,\mathbb{Z})$								×						×				
$(\mathbb{Z},\mathbb{Z}^2\oplus\mathbb{Z}_2,\mathbb{Z}\oplus\mathbb{Z}_2,0)$														×		×	×	×
$(\mathbb{Z},\mathbb{Z}^2\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2,\mathbb{Z}\oplus\mathbb{Z}_2,0)$																	×	
$(\mathbb{Z},\mathbb{Z}^2\oplus\mathbb{Z}_4,\mathbb{Z}\oplus\mathbb{Z}_2,0)$																		×
$(\mathbb{Z},\mathbb{Z}^2\oplus\mathbb{Z}_7,\mathbb{Z}^2,\mathbb{Z})$																	×	
$(\mathbb{Z},\mathbb{Z}^3,\mathbb{Z}^2\oplus\mathbb{Z}_2,0)$																		×
$(\mathbb{Z},\mathbb{Z}^3,\mathbb{Z}^3,\mathbb{Z})$											×	×	×	×	×	×	×	×
$(\mathbb{Z},\mathbb{Z}^3\oplus\mathbb{Z}_2,\mathbb{Z}^2\oplus\mathbb{Z}_2,0)$																	×	
$(\mathbb{Z},\mathbb{Z}^4,\mathbb{Z}^4,\mathbb{Z})$																×		
$(\mathbb{Z},\mathbb{Z}^5,\mathbb{Z}^5,\mathbb{Z})$												×						
$(\mathbb{Z},\mathbb{Z}^6,\mathbb{Z}^5\oplus\mathbb{Z}_2,0)$																×		
$(\mathbb{Z},\mathbb{Z}^6,\mathbb{Z}^6,\mathbb{Z})$																×		
$(\mathbb{Z},\mathbb{Z}^7,\mathbb{Z}^6\oplus\mathbb{Z}_2,0)$														×				
$(\mathbb{Z},\mathbb{Z}^7,\mathbb{Z}^7,\mathbb{Z})$														×				
$(\mathbb{Z},\mathbb{Z}^9,\mathbb{Z}^8\oplus\mathbb{Z}_2,0)$																×		
$(\mathbb{Z},\mathbb{Z}^{12},\mathbb{Z}^{11}\oplus\mathbb{Z}_2,0)$																		×
$(\mathbb{Z},\mathbb{Z}^{12},\mathbb{Z}^{12},\mathbb{Z})$																	×	×

It is interesting to see that some of the homological types of the complexes do not occur for certain integers. Especially, if n is a prime number, the number of homologically distinct complexes seems to be limited. In particular, we believe the following to be true.

Conjecture 2.3. *Let M be a combinatorial 3-manifold with cyclic automorphism group homeomorphic to the total space of the orientable sphere bundle over the circle $S^2\times S^1$. Then M has an even number of vertices.*

3 Infinite series of combinatorial manifolds

It has always been interesting to see, how cyclic combinatorial manifolds or other highly symmetric complexes can be generalized to a whole family of objects sharing this property. See for example the infinite series of the so-called Altshuler tori with dihedral automorphism group [1, Theorem 4], a family of several infinite series of combinatorial manifolds by Kühnel and Lassmann in [13], a neighborly infinite series of the 3-dimensional Klein bottle in [12] and a neighborly infinite series of the 3-torus in [4].

In the case of combinatorial complexes with cyclic automorphism group, a generalization of a given complex to an infinite series of such triangulations with increasing number of vertices seems somewhat natural. One way to see this uses slicings of combinatorial 3-manifolds as described in [23, Section 4.2]. The idea is to generalize a slicing of a combinatorial 3-manifold extending the cyclic symmetry. More generally, in the case of a cyclic combinatorial 3-manifold represented by a set of difference cycles, there is a simple combinatorial condition whether a given triangulation can be generalized to an infinite family of cyclic complexes or not.

Theorem 3.1. *Let $M = \{d_1, \ldots, d_m\}$ be a combinatorial 3-manifold with n vertices, represented by m differnce cycles $d_i = (a_i^0 : \ldots : a_i^3)$, $1 \le i \le m$. Without loss of generality let us assume that $a_i^3 \ge a_i^j$ for all $1 \le i \le m$, $0 \le j \le 2$.*

Then the complex $M_k = \{d_{1,k}, \ldots, d_{m,k}\}$ with $d_{i,k} = (a_i^0 : \ldots : a_i^3 + k)$, $1 \le i \le m$, is a combinatorial manifold for all $k \in \mathbb{N}_0$ if and only if $a_i^3 > a_i^0 + \ldots + a_i^2$ for all $1 \le i \le m$.

In order to prove Theorem 3.1 let us first take a look at a few lemma.

Lemma 3.2. *Let $(a_0 : \ldots : a_d)$ be a difference cycle of dimension d on n vertices and $1 \le k \le d+1$ the smallest integer such that $k \mid (d+1)$ and $a_i = a_{i+k}$, $0 \le i \le d-k$. Then $(a_0 : \ldots : a_d)$ is of length $\sum_{i=0}^{k-1} a_i = \frac{nk}{d+1}$.*

Proof. We set $m := \frac{nk}{d+1}$ and compute

$$\begin{aligned} \left\langle 0+m, a_0+m, \ldots, (\Sigma_{i=0}^{d-1} a_i) + m \right\rangle &= \left\langle \Sigma_{i=0}^{k-1} a_i, \Sigma_{i=0}^{k} a_i, \ldots, \Sigma_{i=0}^{d-1} a_i, 0, a_1, \ldots, \Sigma_{i=0}^{k-2} a_i \right\rangle \\ &= \left\langle 0, a_0, \ldots, \Sigma_{i=0}^{d-1} a_i \right\rangle \end{aligned}$$

(all entries are computed modulo n). Hence, for the length l of $(a_0 : \ldots : a_d)$ we have $l \le \frac{nk}{d+1}$ and since k is minimal with $k \mid (d+1)$ and $a_i = a_{i+k}$, the upper bound is attained. □

Lemma 3.3. *Let $(M_k)_{k \in \mathbb{N}_0}$ be an infinite series of cyclic combinatorial 3-manifolds with $n+k$ vertices represented by the union of m difference cycles of full length, that is, the length of the difference cycles equals the number of vertices $n+k$ of the complex. Then we have for the f-vector of the series*

$$f(\mathrm{lk}_{M_0}(0)) = f(\mathrm{lk}_{M_k}(0)) = (2m+2, 6m, 4m)$$

for all $k \in \mathbb{N}_0$. In particular, the number of vertices of $\mathrm{lk}_{M_k}(0)$ does not depend on the value of k.

Proof. Since M_k is the union of m difference cycles of full length, we have for the number of tetrahedra $f_3(M_k) = m(n+k)$ for all $k \in \mathbb{N}_0$. Furthermore, as M_k is cyclic, all vertices are contained in the same number of tetrahedra which has 4 vertices. By the fact that any facet of $\mathrm{lk}_{M_k}(0)$ corresponds to a facet in M_k containing 0 it follows that for the number of triangles of the link $f_2(\mathrm{lk}_{M_k}(0)) = \frac{4m(n+k)}{n+k} = 4m$ holds, which is independent of k. Since for all $k \in \mathbb{N}_0$ M_k is a combinatorial 2-sphere, all edges of lk_{M_k} lie in exactly two triangles, hence $f_1(\mathrm{lk}_{M_k}(0)) = 6m$. Finally, the Euler characteristic of the 2-sphere is 2, and by the Euler-Poincaré formula we have $f_0(\mathrm{lk}_{M_k}(0)) = 2m + 2$. □

Let us now come to the proof of Theorem 3.1.

Proof. Now let $M = \{d_1, \ldots, d_m\}$ be a combinatorial 3-manifold with n vertices, represented by m differnce cycles $d_i = (a_i^0 : \ldots : a_i^3)$, $1 \leq i \leq m$, such that $a_i^3 > a_i^0 + \ldots + a_i^2$ for all $1 \leq i \leq m$.

For the link of vertex 0 in M we then have:

$$\mathrm{lk}_M(0) \;=\; \bigcup_{i=1}^{m} \bigcup_{j=-1}^{2} \langle -\sum_{k=0}^{j} a_i^k, \ldots, -a_i^j, a_i^{j+1}, \ldots \sum_{k=j+1}^{2} a_i^k \rangle \tag{3.1}$$

which has to be a triangulated 2-sphere, as M is a combinatorial 3-manifold. Since $a_i^3 > \frac{n}{2} > a_i^0 + \ldots + a_i^2$ for all $1 \leq i \leq m$, the vertices $v_j \in \{0, \ldots n-1\}$ of $\mathrm{lk}_M(0)$ can be mapped to the vertices of $\mathrm{lk}_{M_k}(0)$, $k \in \mathbb{N}_0$, as follows:

$$v_j \mapsto \begin{cases} v_j & \text{if } v_j < \frac{n}{2} \\ v_j + k & \text{if } v_j \geq \frac{n}{2}. \end{cases}$$

This yields a combinatorial isomorphism between $\mathrm{lk}_M(0)$ and $\mathrm{lk}_{M_k}(0)$. Since M and M_k are cyclic, all vertex links are isomorphic. Altogether it follows that M_k is a combinatorial manifold for all $k \in \mathbb{N}_0$.

This part of the proof can be generalized to combinatorial d-manifolds, d arbitrary, see Theorem 3.7.

Conversely, let $M = \{d_1, \ldots, d_m\}$ be a combinatorial 3-manifold with n vertices, represented by m differnce cycles $d_i = (a_i^0 : \ldots : a_i^3)$, $1 \leq i \leq m$, such that $M_k = \{d_{1,k}, \ldots, d_{m,k}\}$ with $d_{i,k} = (a_i^0 : \ldots : a_i^3 + k)$, $1 \leq i \leq m$, is a combinatorial manifold for all $k \in \mathbb{N}_0$. Now, let us suppose that there exist a $\tilde{k} \in \mathbb{N}_0$ such that $a_i^3 + \tilde{k} = a_i^0 + \ldots + a_i^2$ for one difference cycle d_i and $a_j^3 + \tilde{k} \geq a_j^0 + \ldots + a_j^2$ for all other $1 \leq j \leq m$. Since $a_j^3 + \tilde{k} \geq a_j^0 + \ldots + a_j^2$ and $a_j^l > 0$ for all $1 \leq j \leq m$, $0 \leq l \leq 3$, it follows by Lemma 3.2 that all difference cycles of $M_{\tilde{k}}$ and $M_{\tilde{k}+1}$ have full length. By Lemma 3.3 it now follows that the links of vertex 0 in $M_{\tilde{k}}$ and $M_{\tilde{k}+1}$ have the same f-vector. On the other hand, since $a_i^3 + \tilde{k} = a_i^0 + \ldots + a_i^2$ but $a_j^3 + \tilde{k} + 1 > a_j^0 + \ldots + a_j^2$ for all $1 \leq j \leq m$, we can see by looking at the vertices of $\mathrm{lk}_{M_{\tilde{k}}}(0)$ that $\mathrm{lk}_{M_{\tilde{k}+1}}(0)$ has to have strictly more vertices than the link of vertex 0 in $M_{\tilde{k}}$. This is a contradiction to Lemma 3.3. □

Remark 3.4. Theorem 3.1 shows, how a single cyclic combinatorial 3-manifold can be extended to an infinite number of combinatorial 3-manifolds by adding an arbitrary positive integer to the largest entry in every difference cycle. More generally, we will talk about *infinite series of cyclic combinatorial d-manifolds* whenever the infinite family of complexes is constructed by adding multiples of a positive integer $k \in \mathbb{N}$ to certain entries of the difference cycles of a combinatorial d-manifold M of arbitrary dimension d. In contrast to that, in Section 4 we will look at an infinite series with an increasing number of difference cycles. Hence, infinite series of combinatorial d-manifolds can be defined in various ways. As a consequence, in every context attention has to be payed what exactly is meant by an infinite series of combinatorial manifolds.

In the following, we will require an infinite series of cyclic combinatorial manifolds to start with the smallest complex that is a combinatorial manifold, that is, the complex M_{-1} must not be a combinatorial manifold.

Corollary 3.5. *Let* $(M_k)_{k\in\mathbb{N}_0}$ *be an infinite series of cyclic combinatorial* 3*-manifolds such that* M_{-1} *is not a combinatorial manifold, then* M_0 *has an odd number of vertices.*

Proof. This follows immediately by the fact, that $\Delta_j := a_j^d - a_j^0 - \ldots - a_j^{d-1} > 0$ for all $1 \leq j \leq m$ in M_0. If the mimimum over all Δ_j, $1 \leq j \leq m$, is greater than 1, M_{-1} is a combinatorial 3-manifold by Theorem 3.1 and M_0 is not the smallest member of that infinite series. Hence, $\Delta_i = 1$ for some $1 \leq i \leq m$ and $n = 2a_i^d + 1$. □

Another direct consequence from the classification and Theorem 3.1 is the following result.

Corollary 3.6. *There are exactly* 396 *combinatorially distinct dense infinite series of combinatorial* 3*-manifolds starting with a triangulation with less than* 23 *vertices.*

So far, we just considered infinite series of cyclic combinatorial manifolds that have members for all integers $n \geq n_0$ for n_0 sufficiently large. However, the notion of an infinite series of combinatorial manifolds as described in Remark 3.4 is more general. In fact, there are other (weaker) formulations of infinite series of cyclic combinatorial d-manifolds: In the following, we will call a series N_k *of order* l, $l \in \mathbb{N}$, if there exist an integer $n_0 \in \mathbb{N}$ such that there are triangulations with $n = n_0 + k \cdot l$ vertices in N_k for all $k \in \mathbb{N}$. It will usually be denoted as $(N_k)_{k \in l\mathbb{N}_0}$. The case $l = 1$ contains all other cases. It coincides with the previously described series and will be referred to as a *dense* series.

There is an analogue to the first half of Theorem 3.1 for infinite series of combinatorial d-manifolds of order l, $1 < l \leq d$, which can be formulated as follows.

Theorem 3.7. *Let* $N = \{d_1, \ldots, d_m\}$ *be a combinatorial d-manifold with n vertices, represented by m differnce cycles* $d_i = (a_i^0 : \ldots : a_i^d)$, $1 \leq i \leq m$.

Then there is a combinatorial d-manifold $N_k = \{d_{1,k}, \ldots, d_{m,k}\}$ *with* $d_{i,k} = (\tilde{a}_{i,k}^0 : \ldots : \tilde{a}_{i,k}^d)$, $1 \leq i \leq m$, *for all* $k \in \mathbb{N}_0$, *if for all* $1 \leq i \leq m$ *there exist a partition* $(l_0, \ldots, l_d)$ *of* $l \in \mathbb{N}$ *allowing zero entries such that*

$$\frac{(l_j + 1)n}{l + 1} > a_i^j > \frac{l_j n}{l + 1},$$

$0 \leq j \leq d$. *In this case we have* $\tilde{a}_{i,k}^j = a_i^j + l_j k$, $0 \leq j \leq d$.

Proof. The proof is completely analogue to the one of the first part of Theorem 3.1. Here, too, we look at a relabeling of the vertices of the link $\mathrm{lk}_N(0)$ in order to transform it to $\mathrm{lk}_{N_k}(0)$.

The relabeling is given by

$$v_j \mapsto v_j + \left\lfloor \frac{(d+1)v_j}{n} \right\rfloor k.$$

The first half of Theorem 3.1 corresponds to the case $d = 3$ and $l = 1$. □

Theorem 3.7 defines series of order l, $1 \leq l \leq d$, by a purely combinatorial criterion. Since all dense series contain series of order l, the following characterisation of higher order series is interesting.

Lemma 3.8. *Let $(N_k)_{k \in l\mathbb{N}}$ be an infinite series of combinatorial d-manifolds of order l, $1 \leq l \leq d$, with $n + lk$ vertices given by a partition $(l_0, \ldots, l_d)$ of l, $l_j \geq 0$, and $N_k = \{(\tilde{a}^0_{1,k} : \ldots : \tilde{a}^d_{1,k}), \ldots, (\tilde{a}^0_{m,k} : \ldots : \tilde{a}^d_{m,k})\}$ such that $\tilde{a}^j_{i,k} = a^j_i + l_j k$, $0 \leq j \leq d$, where the a^j_i, $1 \leq i \leq m$, $0 \leq j \leq d$, denote the entries of the difference cycles of N_0. Then all but finitely many members of $(N_k)_{k \in l\mathbb{N}}$ are contained in a dense series, if l is a unit in $\mathbb{Z}_n$.*

Proof. By multiplying N_k by l we get $lN_k = \{(la^0_{1,k} : \ldots : la^d_{1,k}), \ldots, (la^0_{m,k} : \ldots : la^d_{m,k})\}$. Hence, we have $la^j_{i,k} = la^j_i + ll_j k = la^j_i - l_j n$ which is independent of k. By adding $n + lk$ to each of the a^d_i, $1 \leq i \leq m$, we have $\sum_{j=0}^{d} l\tilde{a}^j_{1,k} = n + lk$.

Now, if $k = 0$, N_0 has n vertices, and l is a unit in $\mathbb{Z}_n$, the multiplied complex lN_0 is a combinatorial manifold and, thus, all differences of lN_0 are non-zero. Since, in lN_k, only $a^d_{i,k}$ depends on k it follows, that for $k \geq k_0$ sufficiently large we can i) rearrange all differences such that all differences are greater than zero and ii) Theorem 3.7 in the case $l = 1$ can be applied. Hence, all N_k, $k \geq k_0$, are contained in an infinite dense series of combinatorial d-manifolds. □

Corollary 3.9. *Let $(N_k)_{k \in 2\mathbb{N}}$ be an infinite series of cyclic combinatorial d-manifolds of order 2, which is not contained in a dense series. Then the number of vertices of $\mathbb{N}_0$ has to be even.*

Proof. This follows immediately since 2 is a unit in $\mathbb{Z}_n$ for all $n \equiv 1(2)$. □

Since Theorem 3.7 is valid for arbitrary dimensions, an extended classification of cyclic combinatorial manifolds of higher dimensions would certainly lead to further interesting results. However, this is work in progress.

4 An infinite series of neighborly lens spaces of varying topological types

All infinite series described in Section 3 have a constant number of difference cycles. Hence, by Lemma 3.3, at most one member of the series can be 2-neighborly. In particular, infinite series of neighborly cyclic combinatorial 3-manifolds must consist of an increasing number of difference cycles.

Moreover, even if we consider all known neighborly series as well, only few topologically distinct 3-manifolds occur in these series. For example, there are a lot of series known with members of type $S^2 \times S^1$ or $S^2 \dot{\times} S^1$ (see [12] or [23, Section 4.2]), S^3 (the the boundary of the cyclic 4-polytopes), $\mathbb{T}^3$ or $\mathfrak{B}_2$ (see [4], [13] or series number 17 from Corollary 3.6, `SCSeries(17,k)` in `simpcomp`) or the series with number 30, 42 and 356 from Corollary 3.6 (`SCSeries(30,k)`, `SCSeries(42,k)` and `SCSeries(356,k)` in `simpcomp`) which contain a few more combinatorial 3-manifolds and up to three distinct topological types per series.

Additionally, series with infinitely many topologically distinct members exist – but the members have increasing dimension (for example the simplices Δ^d, the cross polytopes β^d, the boundary of the cyclic polytopes $\delta C(d+1,n)$, the infinite series of d-tori $\mathbb{T}^d$ in [**?**] or the series M^d_k in [13]).

Thus, neighborly series of combinatorial 3-manifolds which additionally have members of many different topological types would be interesting to investigate. Unfortunately, due to the higher complexity, such series are hard to find. However, using the large amount of complexes from the classification described in Section 2, the following infinite series of topologically distinct lens spaces could be constructed.

Theorem 4.1. *The complex*

$$\begin{aligned} L_k &:= \{ (1:1:1:11+4k), (1:2:4:7+4k), (1:4:2:7+4k), (1:4:7+4k:2) \} \\ & \bigcup_{i=0}^{k} \{ (2:5+2i:2:5+4k-2i), (4:2+2i:4:4+4k-2i) \} \end{aligned} \tag{4.1}$$

is a combinatorial 3*-manifold with* $n = 14+4k$*,* $k \geq 0$*, vertices. It is homeomorphic to the lens space* $L((k+2)^2-1, k+2)$*.*

Proof. Obviously, L_k has $n = 14+4k$ vertices. By looking at Figure 4.1 we can verify that the link $\mathrm{lk}_{L_k}(0)$ of vertex 0 in L_k is a triangulated 2-sphere. Hence, as L_k has transitive symmetry it follows immediately that L_k is in fact a combinatorial 3-manifold for all $k \geq 0$. Furthermore, we can see that $\mathrm{lk}_{L_k}(0)$ has $13+4k$ vertices and thus L_k is 2-neighborly. To determine the exact topological type of L_k we will proceed as follows:

1. For all $k \geq 0$, determine a Heegaard splitting $T^-_k \cup_{S_k} T^+_k$ of L_k of genus 1,
2. draw the center torus S_k of the splitting as a slicing (see Figure 4.2),
3. choose a base $H_1(\partial T^-_k) = \langle \alpha^-_k, \beta^-_k \rangle$ of the 1-homology of the boundary of the lower solid torus T^-_k such that $H_1(T^-_k) = \langle \beta^-_k \rangle$,
4. do the same for the upper solid torus T^+_k such that $H_1(\partial T^+_k) = \langle \alpha^+_k, \beta^+_k \rangle$ and $H_1(T^+_k) = \langle \beta^+_k \rangle$,
5. determine the homological type of α^-_k in $H_1(\partial T^+_k)$ – by construction this will be a torus knot which will determine the topological type of L_k.

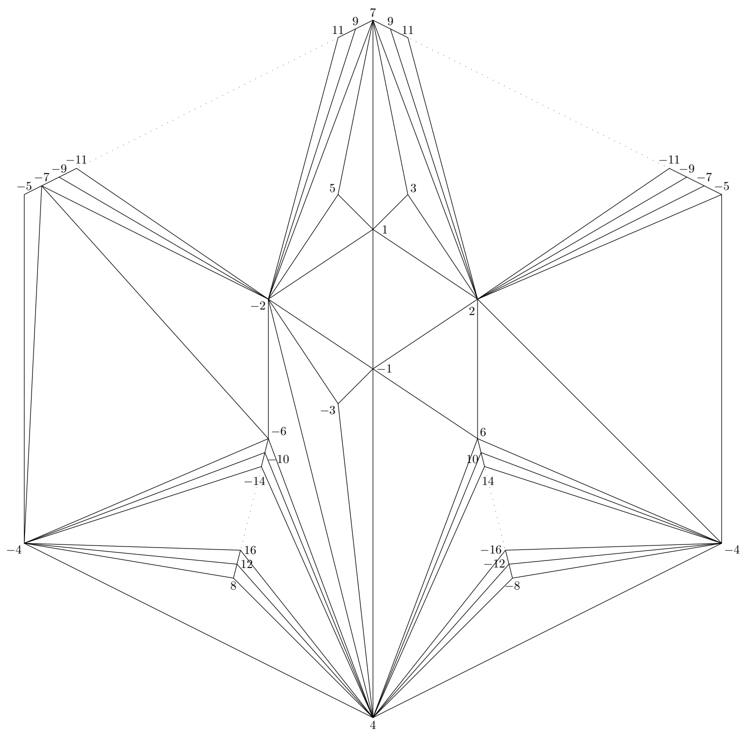


Figure 4.1: Link of vertex 0 of L_k – a triangulated 2-sphere with $13 + 4k$ vertices.

1. For all $k \geq 0$, the span of the even labeled vertices $T_k^- := \text{span}(\{0, 2, \ldots, n-1\})$ as well as the span of the odd labeled vertices $T_k^+ := \text{span}(\{1, 3, \ldots, n\})$ (which is combinatorially isomorphic to T_k^- by the cyclic symmetry) form a solid torus and hence the slicing between the odd and the even vertices $S_k := S_{(\{0,2,\ldots\},\{1,3,\ldots\})}(L_k)$ is a triangulated torus.

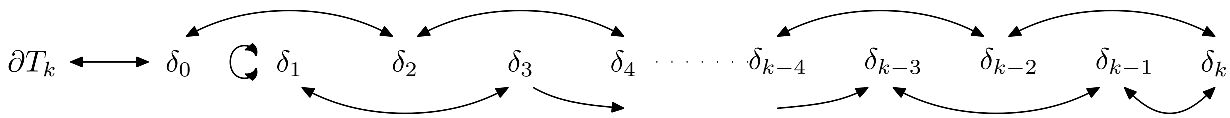
To see this note that T_k^- together with T_k^+ are exactly the difference cycles

$$T_k^- \cup T_k^+ = \bigcup_{i=0}^{k} \{ (4 : 2 + 2i : 4 : 4 + 4k - 2i) \} \subset L_k.$$

Since the gcd of 4, $2 + 2i$ and $4 + 4k - 2i$, $0 \leq i \leq k$, is 2 for all $k \geq 0$, both T_k^- and T_k^+ are disjoint but connected and we have

$$T_k^- \cong T_k^+ \cong \bigcup_{i=0}^{k} \{ (2 : 1 + i : 2 : 2 + 2k - i) \} =: T_k.$$

For $k = 0$ we have $T_0 = \{(1 : 1 : 1 : 4)\} \cong B^2 \times S^1$. Now let $k \geq 1$. T_k consists of $k+1$ difference cycles and we will note $\delta_i := (2 : 1 + i : 2 : 2 + 2k - i)$. δ_i shares two triangles per tetrahedron with δ_{2+i}, $0 \leq i \leq k - 2$, δ_{k-1} shares two triangles per tetrahedron with δ_k, $k \geq 1$, δ_1 shares two triangle per tetrahedron with itself and δ_0 shares two triangles per tetrahedron with ∂T_k and hence contains the complete boundary of T_k. Altogether, we have the following collapsing sheme of T_k:



Thus, T_k collapses onto $\delta_1 = (2:2:2:1+2k)$ and since the modulus of δ_1 is odd we have $\delta_1 \cong (1:1:1:4+2k) \cong B^2 \times S^1$. As a direct consequence, $T_k^- \cup_{S_k} T_k^+$ defines a Heegaard splitting of L_k of genus 1 and L_k is homeomorphic to the 3-sphere, $S^2 \times S^1$ or a lens space $L(p,q)$.

2. The center piece of the Heegaard splitting $S_k := S_{(\{0,2,\ldots\},\{1,3,\ldots\})}(L_k)$ is shown in Figure 4.2. It is interesting to see that apart from T_k^- and T_k^+ the difference cycles $(1:2:4:7+4k)$ and $(1:4:2:7+4k)$ are the only ones which do not contain two odd and two even labels per tetrahedron and thus are the only ones which are not sliced by S_k in a quadrilateral. Hence, S_k consists of only $28+8k$ triangles but $(2+k)(14+4k)+7+2k = 4k^2+24k+35$ quadrilaterals. Its complete f-vector is

$$f(S_k) = (4k^2+28k+49, 8k^2+60k+112, (8k+28)\Delta, (4k^2+24k+35)\square).$$

3. and **4.** In order to find a suitable basis of $H_1(\partial T_k^-)$ as indicated above, let us first take a look at ∂T_k^- itself which is shown in Figure 4.3. We choose the Basis of $H_1(\partial T_k^-) = \langle \alpha_k^-, \beta_k^- \rangle$ to be

$$\begin{aligned} \alpha_k^- &= \langle 0,4,8,\ldots,n-6,0 \rangle \\ \beta_k^- &= \langle 0,6,12,18,22,26,\ldots,n-4,0 \rangle \end{aligned}$$

or in the case that $n < 26$ as indicated in Figure 4.3. By construction, α_k^- is contractible in T_k^- and $H_1(T_k^-) = \langle \beta_K^- \rangle$.

For $H_1(\partial T_k^+) = \langle \alpha_k^+, \beta_k^+ \rangle$ we choose analogously

$$\begin{aligned} \alpha_k^+ &= \langle 1,5,9,\ldots,n-5,1 \rangle \\ \beta_k^+ &= \langle 1,7,13,19,23,27,\ldots,n-3,1 \rangle \end{aligned}$$

and hence $H_1(T_k^+) = \langle \beta_K^+ \rangle$.

5. To finish the proof we will express α_k^- in terms of α_k^+ and β_k^+. This is done by a map $\phi : H_1(\partial T_k^-) \to H_1(\partial T_k^+)$ which lifts any path in L_k passing only even labeled vertices (a path in ∂T_k^-) to a homologically equivalent path passing only odd labeled vertices (a path in ∂T_k^+). The image of a path under ϕ can be determined with the help of the slicing S_k. In the case of α_k^- it is the thick line in Figure 4.2 and results in the following path:

$$\begin{aligned} \phi(\alpha_k^-) = \langle\ & n-7, n-9, n-11, \ldots, 9, 7, 1, n-1, n-3, \\ & n-3, n-5, n-7, \ldots, 13, 11, 5, 3, 1, \\ & 1, n-1, n-3 \ldots, 17, 15, 9, 7, 5, \\ & \ldots \\ & n-13, n-15, n-17, \ldots, 3, 1, n-5, n-7 \quad \rangle. \end{aligned} \tag{4.2}$$

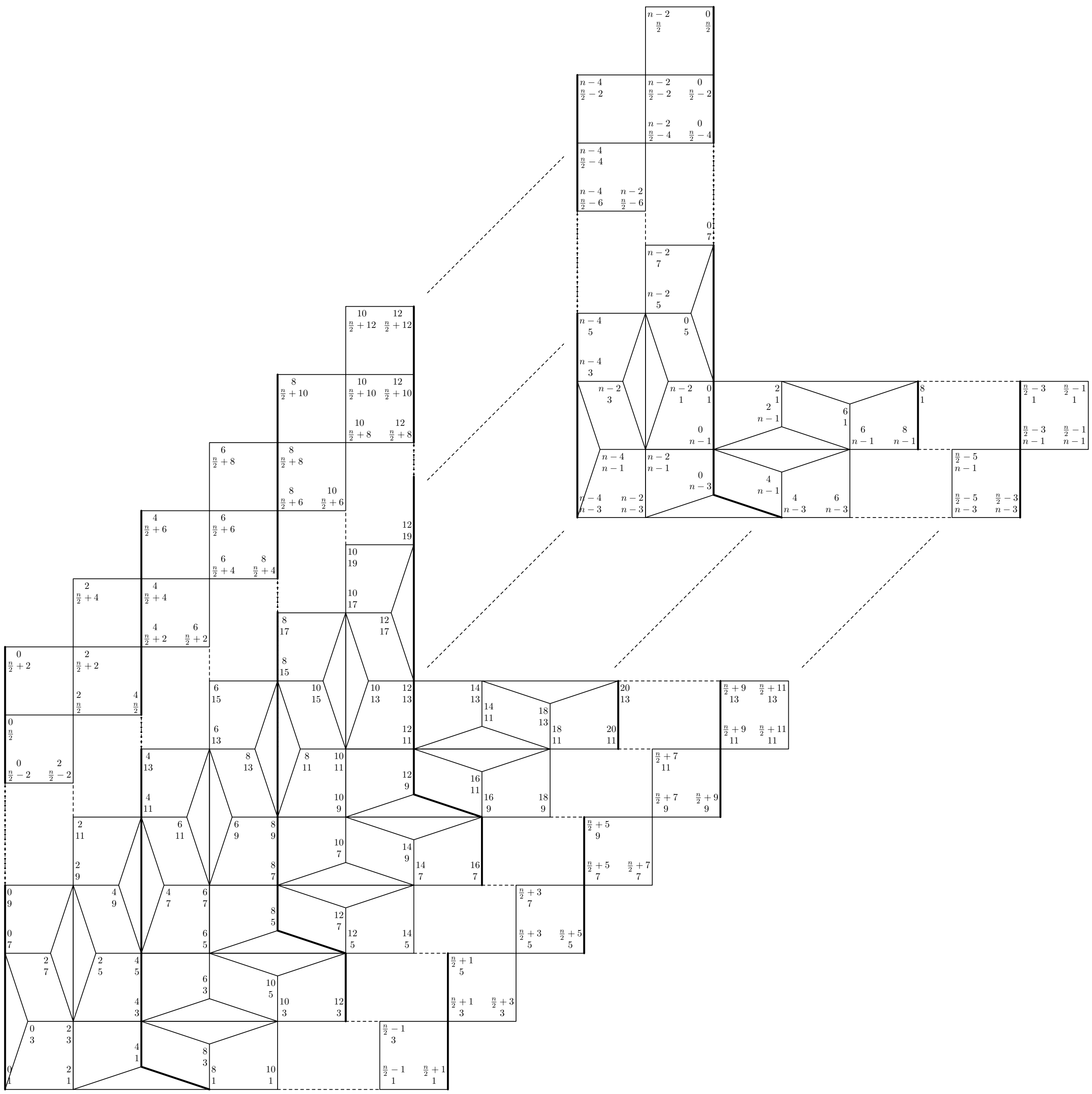


Figure 4.2: Slicing of L_k between the odd labeled and the even labeled vertices – a triangulated torus.

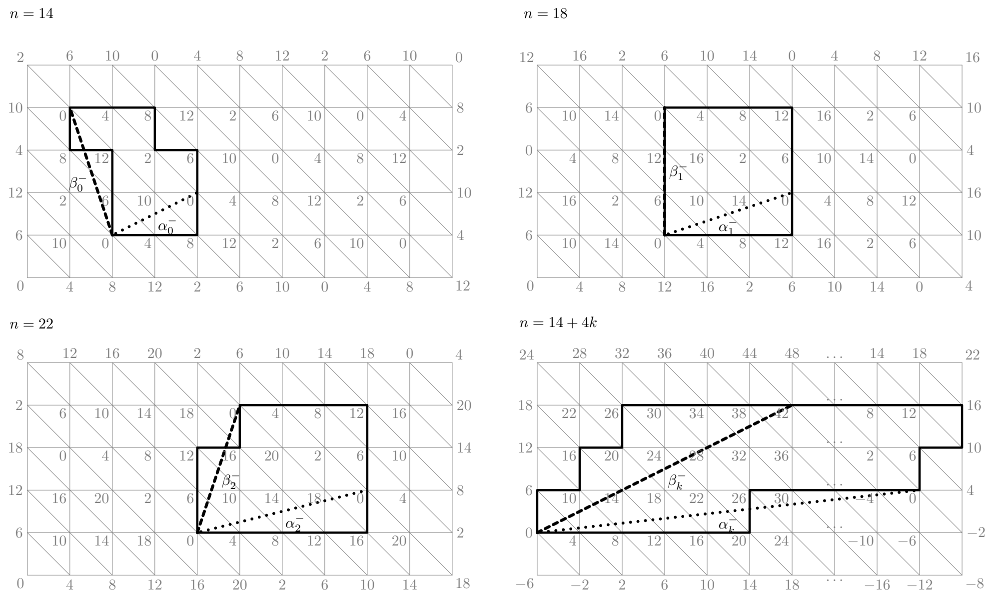


Figure 4.3: Fundamental domain of the boundary of T_k^- together with the basis $\langle \alpha_k^-, \beta_k^- \rangle$ of $H_1(\partial T_k^-)$ for selected values of k and in greater generality.

By taking a closer look to Figure 4.3 we see that all edges of a path of type $\langle s, s-2\rangle$ in both ∂T_k^- and ∂T_k^+ go from the left upper corner of a square of the grid to the lower right corner ($\searrow$) whereas an edge of type $\langle s, s-6\rangle$ is simply going down in the grid ($\downarrow$). Hence, $\phi(\alpha_k^-)$ has $(k+2)(2k+2)+2k+1$ segments of type $\searrow$ and $k+3$ segments of type $\downarrow$ which results in the vector $(2k^2+8k+5, 2k^2+9k+8)$ on the integer grid with basis $(\rightarrow, \downarrow)$ (cf. Figure 4.3 where ∂T_k^+ is obtained from ∂T_k^- by the shift $v \mapsto (v+1) \mod n$ of all vertex labels).

On the other hand, we know that α_k^+ corresponds to the vector $(k+2, -1)$ and β_k^+ to $(k-1, -3)$ on the grid for ∂T_k^+ with basis $(\rightarrow, \downarrow)$. Thus, to express $\phi(\alpha_k^-)$ in terms of α_k^+ and β_k^+ we have to solve the following system of equations:

$$\begin{array}{lrcrcl} \text{I.} & (k+2)q & + & (k-1)p & = & 2k^2+8k+5 \\ \text{II.} & -\quad q & - & 3p & = & 2k^2+9k+8 \end{array} \tag{4.3}$$

which results in the solution

$$q = k^2+3k+1; \quad p = -k^2-4k-3$$

and hence

$$\phi(\alpha_k^-) = (k^2+3k+1)\alpha_k^+ + (-k^2-4k-3)\beta_k^+.$$

Furthermore, note that $L(p, q_1) \cong L(p, q_2)$ if and only if $q_1 \equiv \pm q_2^{\pm 1} \mod p$ from which it follows that

$$K_k \cong L((k+2)^2-1, k+2).$$

□

The series L_k can be modified into a series of 3-spheres which only differs to L_k in the part which is disjoint to slicing. Hence, Theorem 4.1 shows that combinatorial surgery of infinitely many essentially different types can be applied in a setting respecting the cyclic symmetry of the underlying combinatorial manifolds. The following corollary, which is a direct implication of Theorem 4.1, summarizes the findings of this section under a more general point of view.

Corollary 4.2. *There are infinitely many topologically distinct combinatorial (prime)* 3-*manifolds with transitive cyclic automorphism group.*

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